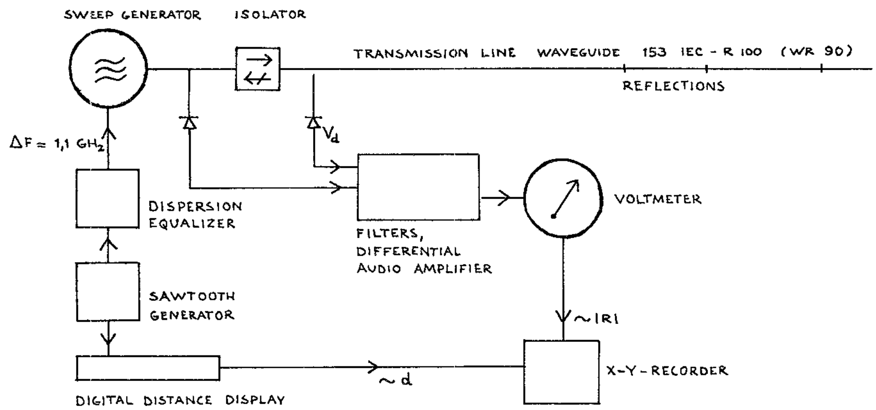

Fig. 1. Block diagram of the distance measuring system.

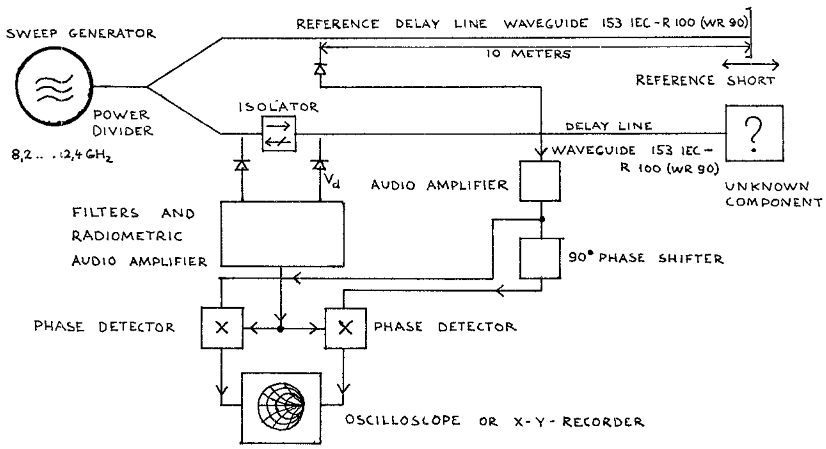

Fig. 2. Block diagram of the Smith chart plotter.

where V_d is the output voltage of the detector, k is a constant, r_n the magnitude of the nth reflection, d_n the distance of the nth reflection, and ϕ_n the phase angle of the nth reflection. f_m is the sawtooth modulation frequency, ΔF the modulation bandwidth, and v the velocity of propagation on the line (v is constant for coaxial lines, but frequency dependent in the case of waveguides; (1) holds, however, for both cases).

Two promising applications of the foregoing principle were developed in detail; the first measuring distance and magnitude of a number of reflections on a transmission line (e.g., an antenna feeder in a microwave relay station), the second measuring magnitude and phase angle of the voltage reflection coefficient of microwave components placed at the end of a delay line.

For the first case an experimental X-band version was built (see Fig. 1). This system proved to have many advantages over a pulse radar of similar performance. It was designed to plot directly reflection magnitude vs. distance. A sensitivity of -70 dB (equivalent to $r=0.0003$ or VSWR 1.0006) referred to a short circuit and an accuracy of ± 1 dB for the magnitude of a reflection and of ± 3 cm for the distance of a reflection were obtained. The useful range of the experimental system was one to twenty meters; however, it could easily be adapted for longer feeder waveguide runs.

For the second case a broadband impedance plotting system was developed, and equally, an experimental X-band version was built (see Fig. 2). This system operates over the whole X-band presenting an oscilloscope or an x-y-ink recorder display of the complex voltage reflection coefficient of an unknown component. The experimental system reached a sensitivity of -60 dB (equivalent to $r=0.001$ or VSWR 1.002) referred to a short circuit and an accuracy of ± 1 dB for the magnitude and of $\pm 5°$ for the phase angle, corresponding to $\pm 0.12r$ within the useful range of $0.001 \leq r \leq 1$. In contrast to earlier developments [1], [2], this system presents the measured impedance (or admittance) directly in a Smith chart display and extends the measurement range about an order of magnitude to smaller reflections.

In both applications a successful attempt has been made to reduce the complexity of the microwave part of the system to a minimum by processing the signals in the audio range. This makes the system easily adaptable to various frequency ranges and waveguide bands.

More detailed analysis is given by Mahle [3].

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Upper and Lower Bounds on the Characteristic Impedance of TEM Mode Transmission Lines

Introduction

A numerical method of obtaining the characteristic impedance of TEM mode transmission lines, which has been described by several authors [1], [2], has the major disadvantage of giving little indication of the resulting error. This correspondence shows how it is possible to extend the finite difference solution of Laplace's Equation to extract an upper and lower bound on the exact solution by using variational formulas. Examples illustrate the high accuracy of solutions obtained with the aid of a digital computer which has been programmed not only to set up and solve the Laplace finite difference equations by systematic over-relaxation but also, at the same time, to compute an upper and a lower bound on the exact solution. Although the method has been used in conjunction with a finite difference solution of Laplace's Equation it can also be used in conjuntion with the Rayleigh Ritz method.

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Method

The finite difference solution of Laplace's Equation gives an approximation to the exact potential function at a finite number of mesh or nodal points within the given boundaries of the transmission line conductors. If a suitable interpolation formula is used to define an approximate, but continuous potential distribution throughout the region, the associated field energy may be calculated. By the Dirichlet principle, or the principle of minimum potential energy, it is known that this energy is greater than the energy E associated with the exact potential function V. It follows that the calculated capacitance per unit length, which is proportional to the field energy, is greater than the exact capacitance per unit length C.

The characteristic impedance Z_0 of a TEM mode transmission line is given by

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{vC}$$

where L is the inductance per unit length and v is the velocity of propagation. A lower bound on the characteristic impedance has thus been obtained.

The dual problem is defined by interchanging electric conductors (short circuit) and magnetic conductors (open circuits) and substituting the reciprocal of the dielectric constant K_e.

$$C' = \frac{2E'}{V_0'^2}.$$

An exact dual potential function V' can be shown to be related to function V of the original problem by the transformation

$$K_e \frac{\partial V}{\partial x} = \frac{\partial V'}{\partial y}, \qquad K_e \frac{\partial V}{\partial y} = -\frac{\partial V'}{\partial x}. \quad (1)$$

Hence

$$E = \frac{\epsilon_0 K_e}{2} \int\int_R \left\{ \left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 \right\} dxdy$$
$$= \frac{\epsilon_0}{2K_e} \int\int_R \left\{ \left(\frac{\partial V'}{\partial y}\right)^2 + \left(\frac{\partial V'}{\partial x}\right)^2 \right\} dxdy$$
$$= E'$$

where E' is the energy in the dual problem and R is the region between the transmission line conductors. Also

$$V_0' = \int_l \frac{\partial V'}{\partial s} ds = \int_l K_e \frac{\partial V}{\partial n} ds = -\frac{Q}{\epsilon_0}$$

where Q is the charge on unit length of the transmission line conductor enclosed by curve l. Thus the exact dual capacitance per unit length is given by

$$C' = \frac{2E'}{V_0'^2} = \frac{2E\epsilon_0^2}{Q^2} = \frac{\epsilon_0^2}{C}.$$

Hence

$$\frac{C'}{\epsilon_0} = \frac{\epsilon_0}{C}, \quad (2)$$

where ϵ_0M8.8542×10^{-12} farad per meter is the permittivity of free space.

If an approximate, but continuous dual potential distribution is found, the associated field energy is greater than the energy E' associated with the exact potential function V'. It follows that the calculated dual capacitance is greater than the exact dual capacitance C'. Hence by (2), a lower bound on the exact capacitance C and thus an upper bound on the characteristic impedance Z_0 have been obtained.

This procedure is analogous to calculating the inductance L of the transmission line with relative permeability K_m equal to $1/K_e$, but its real significance lies in the construction of an approximate, but solenoidal electric field vector, which is to be discussed in a forthcoming paper [4].

The approximate, but continuous dual potential distribution may be obtained directly from the finite difference solution of the original problem with the aid of the transformation (1). This latter method for an upper bound eliminates the need to set up and solve the dual problem, and it was incorporated in the Laplace finite difference computer program described by H. E. Green [3] together with the above method for a lower bound. The following examples illustrate typical results which have been obtained.

Results

a) Square Coaxial Line (Side Length Ratio = 2).

b) Strip Transmission Line ($w/b = 0.8$; $g/b = 3.2$), see Fig. 1.

Problem Number	Number of Nodes in the Finite Difference Net	Characteristic Impedance (Ohms)			
		Lower Bound	Upper Bound	Mean of Upper and Lower	Exact Impedance*
a	441	36.6942	36.9524	36.8229	36.81132
a	1682	36.7636	36.8656	36.8146	
a	6561	36.7921	36.8316	36.8119	
a	Extrapolated to infinity	36.8026	36.8192	36.8109	
b	363	74.4665	77.8513	76.1213	75.9079
b	1365	75.1702	76.8896	76.0202	
b	5289	75.5271	76.3939	75.9580	
b	Extrapolated to infinity	75.6628	76.2072	75.9340	

* The exact impedance was obtained from a conformal transformation.

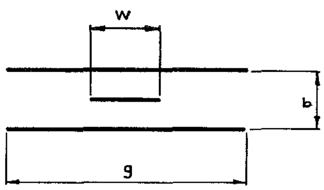

Fig. 1. Strip transmission line.

Conclusion

A method of extending a Laplace finite difference solution to obtain an upper and a lower bound on the characteristic impedance of TEM mode transmission lines has been demonstrated.

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Correction to "General Four-Resonator Filters at Microwave Frequencies"

R. M. Kurzrok, author of the above,[1] has called the following to the attention of the Editor.

On page 296, the first sentence below (4) should have read:

"Letting $|x_v| = 2.75$ and using (4), a theoretical valley insertion loss of 34.4 dB is obtained."

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Manuscript received July 8, 1966.

[1] R. M. Kurzrok, *IEEE Trans. on Microwave Theory and Techniques* (*Correspondence*), vol. MTT-14, pp. 295–296, June 1966.

Addendum to Analysis and Exact Synthesis of Cascaded Commensurate Transmission-Line C-Section All-Pass Networks

In a previous publication [1] a method for the exact synthesis of cascaded commensurate transmission-line C-section all-pass networks was presented. In the general case of n sections, the synthesis procedure requires the solution of a set of n simultaneous linear equations before extracting the even-mode impedances of the coupled lines

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